KnotPlot Exercise #4: Knot Symmetries
Changing the Culture 2000 Workshop

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Knot Symmetries

Many knots have interesting symmetries. We've already seen the torus knots. Those knots had fairly a fairly obvious symmetry to them. Other knots have more sublte symmetries (perhaps, it all depends on how you look at them).

What to do

Start KnotPlot. You first should get a copy of the two page handout with a very quick introduction to KnotPlot. If you're already running KnotPlot, the first thing to do is to type in the command reset all or click on the "Reset" button to get KnotPlot into a "fresh state" to start experimenting (you might not always have to do this, but the exercise may not work as expected if KnotPlot is in some weird state).

Load the Figure-8 knot. Type in: load 4.1
Try rotating it using the virtual trackball. Do you see any symmetry to the knot?

Switch to *orthographic projection*. Sometimes it's easier to see a knot's symmetry by looking at an orthographic projection. Type in ortho 4 4 44 to switch to orthographic (this assumes the view window is still square). Try rotating that Figure-8 knot again. Anything?

Load a new version of the Figure-8. Type in: load 4.1-mirror and convince yourself that this is the same knot. Unrotate the scene by typing untran or clicking on the "untran" button. Now you should see some noticible symmetry.

Rotate it. From an unrotated state, type in rotate x 90. You should see a different symmetry (it helps to decrease the thickness of the tube to about 0.3 using the "cyl-rad" slider). From this rotated state, enter the command rotate y 90
Notice that this rotates the knot into its mirror image! The Figure-8 knot is *equivalent* to its mirror image. Knots having this property are called *achiral* and most knots *do not* have this property (most are inequivalent to their mirror images or are *chiral*, the simplest example is the trefoil).

Untran it again. Type in untran or hit the "Untran" button. From this unrotated state enter the commands:

rotate z -60

rotate y 45

From this new orientation, try the rotation rotate z 180

You should find that this rotates the knot into its mirror image also. This configuration of the Figure-8 knot has *two* rotations about different axes, one by 90° and the other by 180°, each of which maps the knot into it's mirror image!

Try the knot 8_3. Try the exercise using load 8.3-mirror instead of load 4.1-mirror. Again, it's easier to see the symmetry if you reduce the tube thickness to about 0.2 or so.

Higher orders of symmetry. Go to the second demos panel and click on the "D9" button. This knot has a higher order of symmetry than the other two knots we've seen so far. After you run the demo, try load D9 and compare with what you got after clicking the D9 button. These two are the same knot! This is not at all obvious (but it can be proved).